OCR
Oxford Cambridge and RSA

Tuesday 20 June 2017 – Afternoon

A2 GCE MATHEMATICS

4735/01 Probability & Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4735/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 A meteorologist claims that the median daily rainfall in London is 2.2 mm. A single sample sign test is to be used to test the claim, using the following hypotheses:

H_0: a sample comes from a population with median 2.2,

H_1: the sample does not come from a population with median 2.2.

30 randomly selected observations of daily rainfall in London are compared with 2.2, and given a '+' sign if greater than 2.2 and a '–' sign if less than 2.2. (You may assume that no data values are exactly equal to 2.2.) The test is to be carried out at the 5% level of significance. Let the number of '+' signs be k. Find, in terms of k, the critical region for the test showing the values of any relevant probabilities. **[4]**

2 The independent discrete random variables X and Y can take the values 0, 1 and 2 with probabilities as given in the tables.

x	0	1	2
$P(X=x)$	0.5	0.3	0.2

y	0	1	2
$P(Y=y)$	0.5	0.3	0.2

The random variables U and V are defined as follows:

$$U = XY, V = |X - Y|.$$

(i) In the Printed Answer Book complete the table giving the joint distribution of U and V. **[4]**

(ii) Find Cov (U, V). **[5]**

(iii) Find $P(UV = 0 | V = 2)$. **[2]**

3 For events A, B and C it is given that $P(A) = 0.6$, $P(B) = 0.5$, $P(C) = 0.4$ and $P(A \cap B \cap C) = 0.1$. It is also given that events A and B are independent and that events A and C are independent.

(i) Find $P(B | A)$. **[1]**

(ii) Given also that events B and C are independent, find $P(A' \cap B' \cap C')$. **[4]**

(iii) Given instead that events B and C are **not** independent, find the greatest and least possible values of $P(A' \cap B' \cap C')$. **[5]**

4 The heights of eleven randomly selected primary school children are measured. The results, in metres, are

Girls 1.48 1.31 1.63 1.38 1.56 1.57

Boys 1.44 1.35 1.32 1.28 1.27.

(i) Use a Wilcoxon rank-sum test, at the 1% significance level, to test whether primary school girls are taller than primary school boys. **[6]**

(ii) It is decided to repeat the test, using larger random samples. The heights of twenty girls and eighteen boys are measured. Find the greatest value of the test statistic W which will result in the conclusion that there is evidence, at the 1% level of significance, that primary school girls are taller than primary school boys. **[6]**

5 The discrete random variable X is such that $\mathrm{P}(X=x)=\frac{3}{4}\left(\frac{1}{4}\right)^x$, $x = 0, 1, 2, \ldots$.

(i) Show that the moment generating function of X, $\mathrm{M}_X(t)$, can be written as $\mathrm{M}_X(t)=\frac{3}{4-\mathrm{e}^t}$. **[4]**

(ii) Find the range of values of t for which the formula for $\mathrm{M}_X(t)$ in part **(i)** is valid. **[2]**

(iii) Use $\mathrm{M}_X(t)$ to find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **[5]**

6 The continuous random variable Z has probability density function

$$\mathrm{f}(z)=\begin{cases}\frac{4z^3}{k^4} & 0 \leqslant z \leqslant k,\\ 0 & \text{otherwise,}\end{cases}$$

where k is a parameter whose value is to be estimated.

(i) Show that $\frac{5Z}{4}$ is an unbiased estimator of k. **[4]**

(ii) Find the variance of $\frac{5Z}{4}$. **[5]**

The parameter k can also be estimated by making observations of a random variable X which has mean $\frac{1}{2}k$ and variance $\frac{1}{12}k^2$. Let $Y = X_1 + X_2 + X_3$ where X_1, X_2 and X_3 are independent observations of X.

(iii) cY is also an unbiased estimator of k. Find the value of c. **[2]**

(iv) For the value of c found in part **(iii)**, determine which of $\frac{5Z}{4}$ and cY is the more efficient estimator of k. **[4]**

7 The discrete random variable Y has probability generating function $\mathrm{G}_Y(t)=\frac{1}{126}t(64-t^6)\left(1-\frac{t}{2}\right)^{-1}$.

(i) Find $\mathrm{P}(Y=3)$. **[5]**

(ii) Find $\mathrm{E}(Y)$. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			F(9) = 0.0214, F(10) = 0.0494 F(20) = 0.9786, F(19) = 0.9506 $k \le 9$ $k \ge 21$	M1 M1 A1 A1 **[4]**	use of tables, even if 1 tail. look for 2.5% at both ends	Allow even if incorrect eg look for 0.0025. Can be implied by attempt at k≥21.May be by symmetry. N(15,7.5) M1M1A1A0 max.
2	(i)		$V\backslash U$ 0 1 2 4 0 0.25 0.09 0 0.04 1 0.3 0 0.12 0 2 0.2 0 0 0	B1 M1 A1 A1 **[4]**	for correct U and V values attempt to allocate each (x, y) to correct cell. at least 8 cells correct all 12 correct	
	(ii)		E(U) = 0.09 + 0.24 + 0.16 = 0.49 E(V) = 0.42 + 0.4 = 0.82 E(UV) = 2□0.12 = 0.24 Cov(U, V) = “0.24” – “0.49”□“0.82” = -0.1618	M1 M1 M1 M1,A1 **[5]**	their totals ,, ,, their table Allow -0.162	
	(iii)		P(UV = 0∩V = 2)/P(V = 2) = 0.2/0.2 = 1	M1A1 **[2]**	Or verbal explanation.	Either num/denom correct M1, but NOT eg $\frac{0.88\times0.2}{0.2}$
3	(i)		0.5	B1 **[1]**		

Question			Answer	Marks	Guidance	
	(ii)		$0.16 + 0.2 + 0.1 + 0.14 + 0.1 + 0.1 + 0.06$	M2	M1 for at least 4 correct.	0.6+0.5+0.4-0.3-0.24-0.2+0.1=0.86 M2A1. M1 if incorrect coefficient of P($A \cap B \cap C$) used in otherwise correct formula.
			$= 0.86$	A1		
			1 – "0.86"			
			0.14	A1 **[5]**		
	(iii)		greatest : $P(A' \cap B \cap C') = 0.04$, $P(A' \cap B \cap C) = 0.16$ $P(A' \cap B' \cap C) = 0$	M1	for any of these soi eg P(B∩C)=0.26	Greatest: 1-(0.6+0.5+0.4-0.3-0.24-0.26+0.1) = 0.2
			least: $P(A' \cap B \cap C') = 0.2$, $P(A' \cap B \cap C) = 0$ $P(A' \cap B' \cap C) = 0.16$	M1	for any of these soi eg P(B∩C)=0.1	Least 1-(0.6+0.5+0.4-0.3-0.24-0.1+0.1) = 0.04
			greatest 1 – (0.16+0.2+0.04+0.14+0.1+0.16) = 0.2	M1A1	M1 for fully correct method for either.	
			least 1 – (0.16+0.2+0.2+0.14+0.1+0.16) = 0.04	A1 **[5]**		
4	(i)		G: 8 3 11 6 9 10 B: 7 5 4 2 1	B1	Allow 1 set.	
			$R_m = 19$ {5(6+5+1) – 19 = 41} $W = 19$	B1,B1	follow through incorrect ranks for these marks.	
			H_0: two samples come from identical pops H_1: samples do not come from identical pops	B1	or $m_G = m_B$ and $m_G > m_B$	If in words, must say popn.
			"19" > 17, do not reject H_0.	M1		
			insufficient evidence, at 1% level, that p.s. girls are taller than p.s. boys.	A1 **[6]**	cwo. Contextualised, not over-assertive.	
	(ii)		N(351, 1170)	B1,B1		

Question			Answer	Marks	Guidance	
			$\dfrac{W+0.5-351}{\sqrt{1170}}<-2.326$	M1 B1	allow M1 if wrong or no cc. Must be –ve z. 2.326 seen	Allow =.
			$W<270.9$ $W=270$	A1 A1 **[6]**	Allow 271 for this mark, but not from 271.4 allow≤	
5	(i)		$\text{E}(\text{e}^{tx})=\sum_0^\infty \text{e}^{tx}(\tfrac{3}{4})(\tfrac{1}{4})^x$	M1		
			$=\tfrac{3}{4}(\sum_0^\infty(\dfrac{\text{e}^t}{4})^x$	M1	Establish that series is a GP involving t..	
			$\dfrac{3}{4}\left(\dfrac{1}{1-\tfrac{1}{4}\text{e}^t}\right)$	M1	use formula for sum to infinity of GP.	
			$\dfrac{3}{4-\text{e}^t}$ AG	A1 **[4]**		
	(ii)		$\tfrac{1}{4}e^t<1$ $t<\ln 4$	M1 A1 **[2]**	Allow | | for M1, but not A1. t<1.39	
	(iii)		$\text{M}'_X(t)=3\text{e}^t(4-\text{e}^t)^{-2}$ $\text{E}(X)=3\times1\times3^{-2}=\dfrac{1}{3}$	M1 A1	M1 for diffn and sub $t = 0$. cwo	$(4-e^t)^{-2}$ seen
			$\text{M}''_X(t)=\dfrac{3(4-\text{e}^t)^2\text{e}^t+6\text{e}^{2t}(4-\text{e}^t)}{(4-\text{e}^t)^4}$	M1	for diffn using prod/quot rules and sub $t = 0$.	
			$\text{E}(X^2)=\dfrac{3\times3^2\times1+6\times1\times3}{3^4}=\dfrac{5}{9}$	M1	Dep +ve var.	

Question			Answer	Marks	Guidance	
			$\text{Var}(X) = \frac{5}{9} - \left(\frac{1}{3}\right)^2 = \frac{4}{9}$	A1 **[5]**		
6	(i)		$\text{E}(Z) = \int_0^k z\frac{4z^3}{k^4}dz = \left[\frac{4z^5}{5k^4}\right]_0^k = \frac{4k}{5}$ $\text{E}\left(\frac{5Z}{4}\right) = \frac{5}{4}\left(\frac{4k}{5}\right) = k.$	M1A1 M1A1 **[4]**		
	(ii)		$\text{E}(Z^2) = \int_0^k z^2\frac{4z^3}{5k^4}dz = \left[\frac{4z^6}{6k^4}\right]_0^k = \frac{2k^2}{3}$ $\text{Var}(Z) = \frac{2k^2}{3} - \left(\frac{4k}{5}\right)^2 = \frac{2k^2}{75}$ $\text{Var}\left(\frac{5Z}{4}\right) = \frac{25}{16} \times \frac{2k^2}{75} = \frac{k^2}{24}$	M1 M1A1 M1A1 **[5]**		
	(iii)		$\text{E}(cY) = k \quad c(\frac{1}{2}k + \frac{1}{2}k + \frac{1}{2}k) = k$ $c = \frac{2}{3}$	M1 A1 **[2]**		
	(iv)		Var Y$=\frac{3k^2}{12}$ Var $\left(\frac{2Y}{3}\right)=\frac{k^2}{9}$ $\frac{k^2}{24}<\frac{k^2}{9}$ $\frac{5Z}{4}$ more efficient.	B1 B1 M1ft A1 **[4]**		

Question			Answer	Marks	Guidance	
7	(i)		$\frac{1}{126}(64t-t^7)(1+\frac{t}{2}+\frac{t^2}{4}+...)$ P(Y = 3) = coeff of t^3 $=\frac{16}{126}=\frac{8}{63}$	M1A1 M1 M1 A1 **[5]**	M1 for attempt at bin exp, as far as term in t^2 M1 for coeff of t^3 seen or implied Attempt to find term in t^3 Answer, 16/126 oe allow 0.127	
	(ii)		$G_Y'(t)=\frac{1}{126}(64-7t^6)(1-\frac{t}{2})^{-1}$ + $\frac{1}{252}(64t-t^7)(1-\frac{t}{2})^{-2}$ $G_Y'(1)=\frac{40}{21}$	M1 A1 M1A1 **[4]**	attempt at product or quotient rule. sub t = 1 for M1. Allow 1.90	$\text{P(1)}=\frac{64}{126},\text{P(2)}=\frac{32}{126},...\text{P(6)}=\frac{2}{126}$ B1 for Y = 1, 2, ..., 6 B1 for all probs correct M1 for $\sum xp$ used A1 for $\frac{40}{21}$